

CSCI 2244 – Homework 10

Out: **Saturday**, November 23, 2019
Due: **Saturday**, December 7, 2019, 11:59pm

This homework consists of written exercises. You *must* type your solutions. See the “Assignments” section in the syllabus for advice about doing this. You should submit your homework via Canvas. In particular, you should upload a pdf file called:

FirstName_LastName_Homework10.pdf

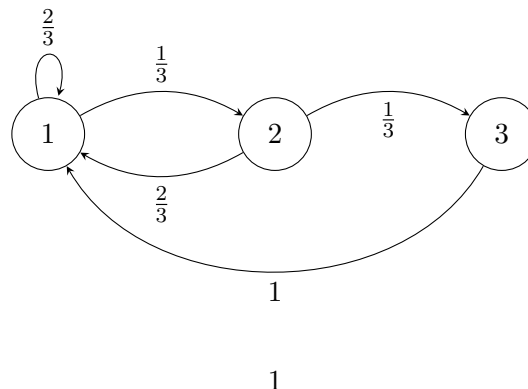
Please use your full first name and last name, as they appear in official university records. The reason for doing so is that the TAs and I must match up these names with the entries in the gradebook.

1 Review

Task 1.1 (10 pts). An author writes a draft of a book that is 200 pages long. Assume that typos occur randomly throughout the book. Each page has a typo with probability 0.01, independently of all the other pages. Let X be the total number of pages with typos.

- What is $\mathbb{E}[X]$?
- What is the probability that more than 3 pages have typos? Write down both an **exact** expression for this probability, and a Poisson approximation.
- The author asks her daughter to proofread the book to find typos. As an incentive, she offers to pay \$0.10 for each page that the daughter finds a typo on. Assume that if a typo occurs on a page, the daughter will find it with probability .8. let Y be the total amount that the author has to pay her daughter. What is $\mathbb{E}[Y]$?

Task 1.2 (10 pts). Consider the following Markov chain:



- (a) What is the mean first passage time from state 1 to state 3?

Task 1.3 (10 pts).

A continuous random variable X has the following density function:

$$f_X(t) = \begin{cases} 0 & t < 0 \\ c(4t^2 + t^3) & 0 \leq t \leq 3 \\ 0 & t > 3 \end{cases}$$

- (a) What must c be equal to, in order for f_X to be a valid density?
(b) The CDF for X has the following form (with part missing):

$$F_X(t) = \begin{cases} 0 & t < 0 \\ \boxed{} & 0 \leq t \leq 3 \\ 1 & t > 3 \end{cases}$$

What is the missing part?

- (c) What is $\text{Var}[X]$?

Task 1.4 (10 pts).

You have two jars:

- Jar A has 3 red marbles and 2 blue marble.
- Jar B has 4 red marbles and 4 blue marbles.

- (a) You reach into Jar A and pull out **two marbles** at once. What is the probability that both marbles are blue?
(b) You put the marbles from the first part of the question back. Then, you close your eyes, randomly put your hand in one of the two jars, each equally likely, and pull out **two marbles** at once. Both turn out to be blue. What is the probability that they came from Jar A?

Task 1.5 (10 pts).

You are conducting interviews for a company. Suppose that each candidate you interview has probability .1 of passing the interview and being hired. You repeatedly interview random candidates one-by-one until you find one that passes the interview.

- (a) What is the expected number of candidates you will interview?
(b) What is the probability that you need to interview 3 or more candidates?
(c) You reimburse candidates' travel expenses. Assume that the i th candidate's travel costs are equal to $100 \cdot Y_i$, where Y_i is a continuous random variable with the uniform distribution on $[0, 1]$. Each of the Y_i is independent of the others, and independent of whether the candidate passes the interview. What is the expected total amount you will pay for all candidates you interview?